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Identification of the structure parameters applying a moving load

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Abstract

An approach to the flexural stiffness identification of a linear structure is proposed. The idea of the presented approach is to transform the dynamical problem into a static one by integrating the input and output signals. The output signal is the structure displacement due to different kinds of loads such as a pulse acting at a given point, moving a load of deterministic or random type. The obtained solution for the one-point force can be easily generalized to a set of point forces, which can be a model of the pressure of vehicle axes. The presented method can be applied to the identification of structure parameters of bridges. It allows also to take into account some stochastic disturbances following the movement of vehicles through the pavement roughness.

1. Introduction

The identification of the most relevant structural properties was and still is necessary to support the calibration, updating and validation of mathematical and numerical models of the structure used at design and service stages. Also in damage detection techniques the regular identification of modal properties, which is possible thanks to the continuous health monitoring of the structure, plays an important role. Therefore, the problem of parameter estimation and system identification, especially as applied to structural engineering, has been a subject of investigations for many scientists for many years and different techniques have been elaborated. A number of results, examples and applications of parameter estimation and system identification techniques elaborated to 1986 are described in Ref. [1]. In the structural identification, Wadia-Fascetti and others [2] have proposed the following repeated six-step methodology:

Step 1: A-priori models development based on the knowledge about the systems.

Step 2: Experiment design, which requires selecting the inputs (loads, excitation, and temperature) and sensors.

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Step 3: Full-scale tests, which include both instrumented monitoring (IM) for static tests and multi-reference impact tests (MRIT) for modal tests. The results from both static and modal tests can be used to verify the different testing methods.

Step 4: Data processing. In this step the data obtained from the experiment must be processed for use in the parameter estimation module such that the quality of the data is maintained.

Step 5: Model calibration and parameter estimation involve identifying critical geometric parameters using the processed static data from Step 4.

Step 6: Utilization of the calibrated models includes obtained interpretations in a form that is useful to other researchers and practitioners.

In general, the identification techniques can be divided into two groups [1,3,4]. The first group is based on the continuous model of the structure, the exact solution and system identification theory, such as the time-domain method (TDM) and the frequency time-domain method (FTDM). The modal superposition technique is applied to decouple the equation of motion with the subsequent solution using the optimization scheme, for example genetic algorithms or evolutionary algorithms [5,6]. The second group of methods is based on discrete models of the structure and finite element formulation, such as the interpretive method (ITM), the state-space approach [7], etc.

Another type of identification techniques division is related to the objective of the identification, which can be the modal parameters of the structure such as the frequencies, damping ratios and mode shapes [8–10] or forces acting on the structure such as moving load [3,4,6,7,11–15] or impact load [14,16–18]. In the experimental modal parameter identification of civil engineering structures, three types of structural dynamic testing can be distinguished [10]:

(1) *force vibration testing*—the structure is excited by artificial means such as shakers or drop weight,
(2) *free vibration testing*—excitation by a sudden dropping of a load on the structure,
(3) *ambient vibration testing*—uses the disturbances inducted by traffic or wind as natural or environmental excitations.

In some cases it is either impractical or impossible to use artificial inputs to excite the system, so natural excitation must be measured along with the system response to assess the dynamic characteristics [19]. Therefore in recent years, many authors have investigated both the problem of load identification (moving load and impact load) and modal structure parameters identification under operational conditions and have applied it to damage detection [4,10,19–21]. Alvin et al. [19] focused their considerations on the state-space oriented system identification theory as specialized to structural dynamics governing the equations of motion. The authors have applied wavelet transformation techniques for extracting impulse response functions, used various input–output combinations for multi-input and multi-output problems, robust ways for identifying both proportional and non-proportional damping parameters, and have shown the use of the localized identification theory for damage detection from measured response data. In his paper, an excellent bibliography with 65 references is also given. As mentioned above, identification of the structure parameters applying a moving load has been considered in many papers, among others in Refs. [3,4,6,7,11,12]. A method based on modal superposition and regularization technique developed to identify moving loads on an elastically supported multi-span continuous bridge desk is presented in paper [3]. Law et al. [4] presented a novel moving force and prestress identification method based on the finite element and the wavelet-based methods for a bridge-vehicle system. Jiang et al. [6] described the parameter identification of a vehicle moving on multi-span continuous bridges, taking into account the surface roughness. In paper [7], a method of moving force identification is developed using the dynamic programming technique. The forces are identified in the time domain using recursive formula and responses are reconstructed using the identified forces for comparison. Different aspects of application of moving loads identification through regularization have been presented in papers [11,12].

In this paper, an approach to the flexural stiffness identification of the beam or linear structure is proposed. The idea of the proposed approach lies within a time-domain method and is similar to that presented in paper [22,23]. The theoretical investigations carried out by Langer and Ruta [22] concerned the dynamic identification of the elastic module of layered half-space. The authors have transformed shock impulse and response oscillograms into static substitute and presented them as the Hankel transform of a superficial displacement state. The method proposed is also based on transforming the dynamical problem into a static one by integrating the input and output signals. The output signal is structure displacements due to the different types of loads such as a pulse acting at a given point, or a moving load of deterministic or random type. The solution obtained for the one-point moving force can be generalized to a set of point moving forces, which can be a model of the pressure of vehicle axes. The general difference between the proposed method and that presented by Langer and Ruta [22] concerns the way in which the dynamical problem is transformed into a static one: the authors of [22] have integrated the solution of the equation of motion and in this paper the equation of motion is integrated immediately after applying eigentransformation. The proposed method of the structure parameter identification compared to the idea presented in Ref. [22] has been extended to the case in which the load acting on the structure has a random nature. The investigations have been limited to the load model described by a moving load in which the interaction between the structure and the load is not taken into account.

In the case of interaction between the structure and the load another, more complex procedure should be applied.

The presented method can be used for the identification of structure parameters of bridges. It allows also to take into account some stochastic disturbances following the movement of vehicles through the pavement roughness.

2. Identification of a beam flexural stiffness in the case of load acting at a given point

The main idea of the proposed procedure for the structure parameters identification will be described on a beam vibrating due to a short-lived load (a pulse) acting at a given place (Fig. 1a and b). The beam vibrations for the simply supported beam and for the continuous beam for the loading span are described by the following equation:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + c\dot{w}(x,t) + m\ddot{w}(x,t) = f(t)p(x) \tag{1}$$

where EI denotes the flexural rigidity of the beam, c is the damping coefficient, m is the mass per unit length, and $F(x,\ t) = f(t)p(x)$ denotes the load as a product of the excitation process $f(t)$ acting from 0 to T and the load distribution on the beam's length $p(x)$. In a particular case when the load is a point force, $p(x) = \delta(x-x_0)$.

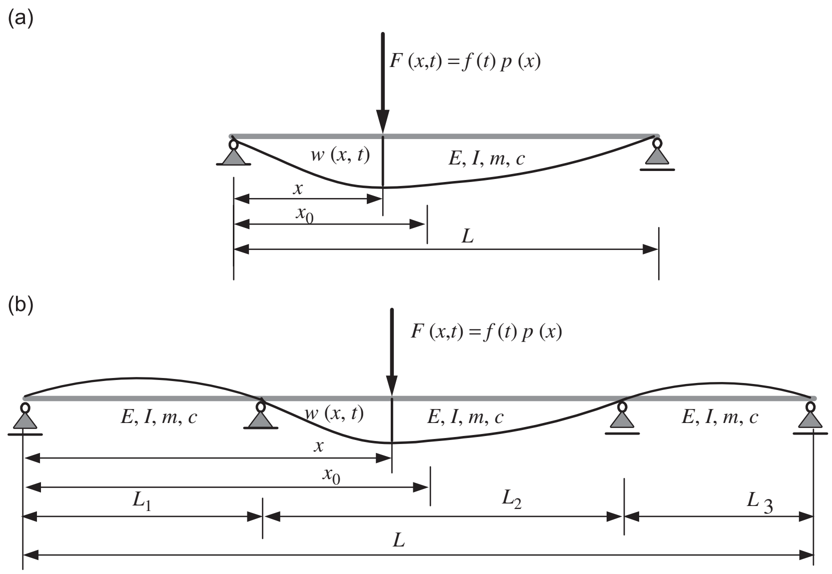

Fig. 1. Scheme of beams vibrating due to a short-lived load acting at a given point.

Let us assume that at the beginning, the beam is in the resting condition, i.e.

$$w(x,0) = 0, \quad \dot{w}(x,0) = 0 \tag{2}$$

Now we will integrate Eq. (1) due to the time within the time interval (0, ∞), keeping in mind that for the load assumed, $\lim_{t\to\infty} w(x,t) = 0$, $\lim_{t\to\infty} \dot{w}(x,t) = 0$. In such a way we obtain the static substitute of the equation of motion (1)

$$EI\, W_0^{IV}(x) = F_0 p(x) \tag{3}$$

where

$$W_0(x) = \lim_{t\to\infty} W(x,t) = \lim_{t\to\infty} \int_0^t w(x,\tau)\,\mathrm{d}\tau \tag{4}$$

$$F_0 = \lim_{t\to\infty} F(t) = \lim_{t\to\infty} \int_0^t f(\tau)\,\mathrm{d}\tau = \lim_{t\to\infty} \int_0^T f(\tau)\,\mathrm{d}\tau \tag{5}$$

and T is the time duration of the load. The shapes of the source functions $w(x, t)$ and $f(t)$ and their integrals $W_0(x)$ and F_0 are shown in Fig. 2.

If the components of the excitation process $f(t)$ and $p(x)$ are known and the beam vibrations process $w(x, t)$ is measured at a given point x, then it is possible to calculate the quantities F_0 from Eq. (5) and $W_0(x)$ from Eq. (4) and then the identification of the beam stiffness EI can be calculated. In particular, when the beam is a simply supported beam assuming that $p(x) = \delta(x-x_0)$, from Eq. (3) and the boundary conditions we obtain

$$W_0(x) = \frac{F_0}{EI}\frac{L^3}{6}\left\{\left(1-\frac{x_0}{L}\right)\frac{x}{L}\left[1-\left(1-\frac{x_0}{L}\right)^2-\left(\frac{x}{L}\right)^2\right]+\left(\frac{x}{L}-\frac{x_0}{L}\right)^3 H(x-x_0)\right\} \tag{6}$$

where the symbol $H(\,)$ denotes the Heaviside unity function.

Let us assume for simplicity that the measure of the vibrations has been carried out at the point $x = x_0$. Hence if the quantities F_0 and $W_0(x_0)$ have been calculated, we obtain

$$EI = \frac{F_0}{W_0(x_0)}\frac{L^3}{3}\left(1-\frac{x_0}{L}\right)^2\left(\frac{x_0}{L}\right)^2 \tag{7}$$

Remark. Every measurement of the beam vibrations and the loading process is performed with some error; therefore the estimate of the constants F_0 and $W_0(x_0)$ might be inaccurate. In order to minimize the influence of these errors, all measurements should be performed several times. For each measurement we determine the

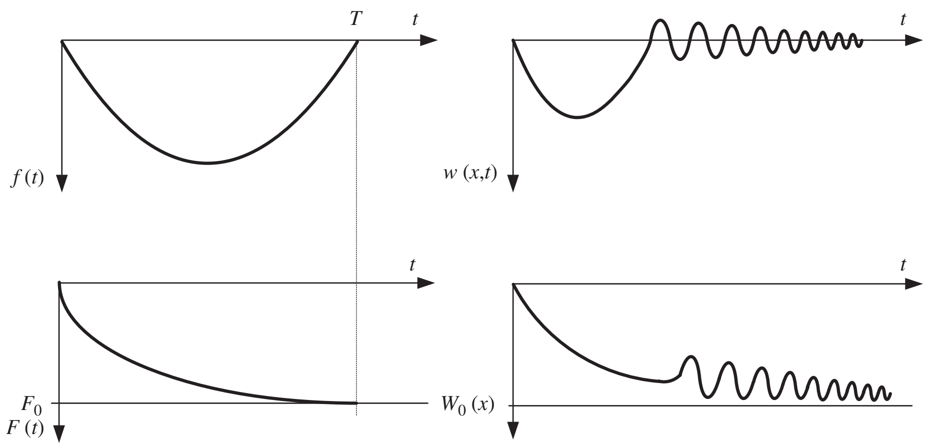

Fig. 2. The shapes of the source functions $w(x, t)$ and $f(t)$ and their integrals $W_0(x)$ and F_0.

beam stiffness by formula (7) and a trustworthy beam stiffness may be defined as an arithmetic mean of so determined values. The beam vibrations should be measured in several points for various placements of the load. The same remark will apply to the considerations presented hereafter.

3. Identification of the beam parameters in the case of a moving load

Now, we will apply, after some modifications, the procedure for identification the beam's parameter presented in a previous chapter to the case when the excitation process is of the type of a moving load. In such a case the load can be treated as a short-lived load at time $T = L/v$. The equation of motion for a beam due to the point force moving along the beam with constant speed v is as follows:

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + c\dot{w}(x,t) + m\ddot{w}(x,t) = P\delta(x - vt), \quad 0 \leqslant t \leqslant \frac{L}{v} \quad 0 \leqslant x \leqslant L \tag{8}$$

and the initial conditions have the form

$$w(x,0) = 0, \quad \left. \frac{\partial w(x,t)}{\partial t} \right|_{t=0} = 0. \tag{9}$$

As is usually done, we transform this partial differential equation into a set of ordinary differential equations, assuming that

$$w(x,t) = \sum_{n=1}^{\infty} y_n(t) W_n(x) \tag{10}$$

where $W_n(x)$ are the eigenfunctions that fulfill the equations given below and appropriate boundary conditions

$$W_n^{IV}(\lambda) - \lambda_n^4 W_n(x) = 0. \tag{11}$$

We obtain

$$\ddot{y}_n(t) + 2\alpha \dot{y}_n(t) + \omega_n^2 y_n(t) = \frac{P}{m\gamma_n^2} W_n(vt) \tag{12}$$

where

$$2\alpha = \frac{c}{m}, \quad \gamma_n^2 = \int_0^L W_n^2(x)\,\mathrm{d}x.$$

The initial conditions take the shape

$$y_n(0) = 0, \quad \dot{y}_n(0) = 0. \tag{13}$$

Now we will transform the dynamic problem into a static one in a similar way as in the previous case.

Let us introduce the following notations:

$$F_n = \int_0^{L/v} W_n(v\tau)\,\mathrm{d}\tau, \quad Y_n(t) = \int_0^t y_n(\tau)\,\mathrm{d}\tau. \tag{14}$$

The magnitude of $Y_n(t)$ converges to a constant magnitude if the time converges to the infinity $t \to \infty$, $Y_n = \lim_{t\to\infty} Y_n(t)$.

From this, it follows that by integrating the equations of motion from zero to infinity we obtain following expression:

$$Y_n = \frac{PF_n}{EI\lambda_n^4\gamma_n^2} = \frac{PF_n}{m\omega_n^2\gamma_n^2}. \tag{15}$$

Let us denote

$$W_0(x) = \sum_{n=1}^{\infty} Y_n W_n(x) = \frac{P}{EI}\sum_{n=1}^{\infty} \frac{F_n}{\lambda_n^4\gamma_n^2} W_n(x) = \sum_{n=1}^{\infty} \frac{PF_n}{m\omega_n^2\gamma_n^2} W_n(x) \tag{16}$$

If we have measured the deflection $w(x, t)$ then we can calculate $W_0(x)$ as

$$W_0(x) = \int_0^\infty w(x,t)\,\mathrm{d}t \tag{17}$$

If we know the load P and have measured the deflection $w(x_0, t)$ at point $x = x_0$ and calculated $W_0(x_0)$ from Eq. (17), then the beam flexural stiffness can be determined from the expression

$$EI = \frac{P}{W_0(x_0)} \sum_{n=1}^{\infty} \frac{F_n}{\lambda_n^4 \gamma_n^2} W_n(x_0). \tag{18}$$

For the simply supported beam we have

$$W_n(x) = \sin\frac{n\pi x}{L}, \quad \int_0^L W_n(x)\,\mathrm{d}x = \frac{L}{2}, \quad \lambda_n^4 = \left(\frac{n\pi}{L}\right)^4,$$
$$F_n = \int_0^{L/v} \sin\frac{n\pi\, vt}{L}\,\mathrm{d}t = \frac{1}{n\pi(v/L)}[1-(-1)^n] \tag{19}$$

and we obtain

$$EI = \frac{2P}{W(x_0)Lv} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{(n\pi/l)^5} \sin\frac{n\pi x_0}{L} \tag{20}$$

Notice that the eigenfunctions and eigenvalues appearing in Eqs. (18) and (20) and determined from Eq. (11) and appropriate boundary conditions depend neither on the stiffness EI nor on the beam mass m. It follows that from the above formulae and from the measured values of $W(x_0)$, one can determine the beam stiffness.

4. Identification of the beam parameters in the case of a random moving load

Let us consider the vibrations of a beam described by the equation of motion (8) in which the moving point force is random and consists of two components, a constant quantity P_0 and the irregular random component $P_s(t)$, i.e.

$$P(t) = P_0 + P_s(t) \tag{21}$$

For simplicity we assume that $E[P_s(t)] = 0$ (this assumption is not necessary).

In such a case Eq. (15) takes the form

$$Y_n = \frac{1}{EI\lambda_n^4\gamma_n^2} \int_0^{L/v} P(\tau) W_n(v\tau)\,\mathrm{d}\tau \tag{22}$$

The second probabilistic moment of the function $W_0(x)$ is described by the expression

$$\begin{aligned} E[W_0^2(x)] &= \frac{1}{(EI)^2} \sum_i \sum_j \frac{1}{\lambda_i^4\lambda_j^4\gamma_i^2\gamma_j^2} \int_0^{L/v}\int_0^{L/v} E[P(\tau_2)P(\tau_2)] W_i(v\tau_1) W_j(v\tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2 \\ &= \frac{1}{(EI)^2} \sum_i \sum_j \frac{1}{\lambda_i^4\lambda_j^4\gamma_i^2\gamma_j^2} \int_0^{L/v}\int_0^{L/v} K_{PP}(\tau_1,\tau_2) W_i(v\tau_1) W_j(v\tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2 \end{aligned} \tag{23}$$

where

$$K_{PP}(\tau_1,\tau_2) = E[P(\tau_1)P(\tau_2)]$$

denotes the correlation function of the random load.

If the expected value of the random component P_s is different from 0, i.e. $E[P_s(t)] \neq 0$, then it influences the correlation function of the random load K_{pp}.

We need to specify how to determine the correlation function $K_{PP}(\tau_1, \tau_2)$ once the realization of $P(t)$ is known from measurements. If $P(t)$ is a realization of a stationary and ergodic process in the time interval

(0, L/v), then the correlation function of this process can be estimated from the formula

$$K_{PP}(\tau) = K_{PP}(\tau_1 - \tau_2) = \frac{v}{L}\int_0^{L/v} P(t)P(t+\tau)\,\mathrm{d}t. \tag{24}$$

If the process $P(t)$ is non-stationary, in order to determine its correlation function we need to apply more complex numerical methods such as wavelet analysis.

On the other hand, the second probabilistic moment can be calculated on the basis of the measured beam's deflections. This allows to identify the flexural rigidity of the beam because Eq. (23) can be presented in the form

$$(EI)^2 = \frac{1}{E[W_0^2(x)]}\sum_i \sum_j \frac{1}{\lambda_i^4 \lambda_j^4 \gamma_i^2 \gamma_j^2}\int_0^{L/v}\int_0^{L/v} K_{PP}(\tau_1, \tau_2) W_i(v\tau_1) W_j(v\tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2. \tag{23a}$$

5. Identification of the linear structures parameters using the stiffness elements method

Let us consider a linear system loaded by a deterministic moving load, the vibrations of which are described by a set of differential equations

$$\mathbf{M}\cdot\ddot{\mathbf{q}}(x,t) + \mathbf{C}\cdot\dot{\mathbf{q}}(x,t) + \mathbf{K}\cdot\mathbf{q}(x,t) = \mathbf{F}(vt), \quad 0 \leqslant t \leqslant \frac{L}{v} \tag{25}$$

and

$$\mathbf{M}\cdot\ddot{\mathbf{q}}(x,t) + \mathbf{C}\cdot\dot{\mathbf{q}}(x,t) + \mathbf{K}\cdot\mathbf{q}(x,t) = 0, \quad t \geqslant \frac{L}{v} \tag{26}$$

where **M**, **C and K** are the mass matrix, the damping coefficients matrix and the stiffness matrix, respectively, $\mathbf{q}(x, t)$ is the vector of generalized coordinates and $\mathbf{F}(vt)$ is the vector of moving load, v denotes the velocity of the moving load and L is the length of the structure.

Similarly as for the beam, we transform the dynamic problem into the static substitute by calculating the integrals of the vectors of general coordinates $\mathbf{q}(x, t)$ and loads $\mathbf{F}(vt)$ (Eqs. (27)) and their limit values (Eqs. (28))

$$\mathbf{Q}(x,t) = \int_0^t \mathbf{q}(x,\tau)\,\mathrm{d}\tau, \quad \mathbf{R}(t) = \int_0^t \mathbf{F}(v\tau)\,\mathrm{d}\tau \tag{27}$$

$$\mathbf{Q}_0(x) = \lim_{t\to\infty}\mathbf{Q}(x,t), \quad \mathbf{R}_0 = \int_0^{L/v}\mathbf{F}(v\tau)\,\mathrm{d}\tau = \lim_{t\to\infty}\mathbf{R}(t) \to 0 \tag{28}$$

After integrating Eq. (24) we obtain

$$\mathbf{K}\cdot\mathbf{Q}_0 = \mathbf{R}_0 \tag{29}$$

If the matrix $\mathbf{R}_0$ and some chosen coordinates of the vector $\mathbf{q}(x, t)$ are known then the vector $\mathbf{Q}_0$ can be determined and hence some parameters of the stiffness matrix **K** can be obtained.

From the theoretical point of view follows that how many general coordinates (displacements) we measure, such many elements of the vector $\mathbf{Q}_0$ can be calculated and such many parameters of the stiffness matrix **K** can be identified or one parameter could be identified repeatedly. Some numerical problems could appear.

The system of matrix Eq. (25) was obtained by the finite elements methods. Notice that it is possible to take into account, among others, variability of the construction rigidity, for example, by assuming a constant rigidity on each finite element.

6. Example—identification of the flexural rigidity of the longitudinal beam of the highway bridge

Usually the bridge test load investigations are due to the static load and therefore the proposed approach for the structure's parameters identification has been tested on the longitudinal and not for the one-span beam of the highway bridge. For such a bridge, the experimental data (vertical displacements) due to moving load were available and could be applied for the identification of flexural rigidity. The considered bridge is a part of the expressway A2 between Poznan and Warsaw, Poland, and consists of five longitudinal steel plane girders of 1800 mm high and distance between them equal to 3700 mm, reinforced concrete plates of 280 mm thickness and steel cross-bars with distance of 7800 mm. The overall length of the bridge is equal to 39.00 + 46.80 + 46.80 + 46.80 + 39.00 = 218.40 m. The total width of the bridge is equal to 17.65 m (roadway, safety zone, sidewalk, barrier). The design characteristics of the girder's cross-section are shown in Fig. 3 and the static model of the longitudinal beam is shown in Fig. 4. The mass per unit length of the bridge girder was equal to 4300 kg/m. During the test load, investigations described in Ref. [24] some dynamical measurements such as the time distribution of the vertical (perpendicular to the bridge axis) and horizontal (parallel to the bridge axis) displacements; the frequencies of the vibrations in the external girder have also been done. Using the laser device NOPTEL OY PSM200, the vertical and horizontal displacements caused by two trucks, each of the total weight equal to 409 kN and the pressure of the rear axis equal to 257 kN, moving one by one and in parallel with different speeds equal to 5, 20 and 40 km/h, have been measured. Some of the vertical displacements (marked by y) and horizontal ones (marked by x) obtained for the middle beam in the middle point of their forth span are presented in Figs. 5 and 6.

The equivalent flexural stiffness of the bridge system (five longitudinal beams, cross-bars, bridge deck) modeled by a continuous beam has been calculated from the design parameters and then identified for the assumed static model applying the technique presented in chapter 3 (Eq. (18)). The calculations have been made for the load caused by moving vehicles, taking into account the lateral distribution on the longitudinal beam for which the displacements have been measured. The eigenfunctions appearing in Eq. (18) have been calculated for the beam of the static model, which is shown in Fig. 4. The equivalent flexural stiffness of the beam calculated from the design parameters is equal to $EI = 20{,}214{,}328{,}317\,\mathrm{N\,m^2}$ and the arithmetic average of identified flexural stiffness after using four eigenvectors for both types of loads and all speeds considered is equal to $EI = 22{,}925{,}044{,}200\,\mathrm{N\,m^2}$. From this test example, no general conclusions can be drawn, because the experimental data have been performed for another use.

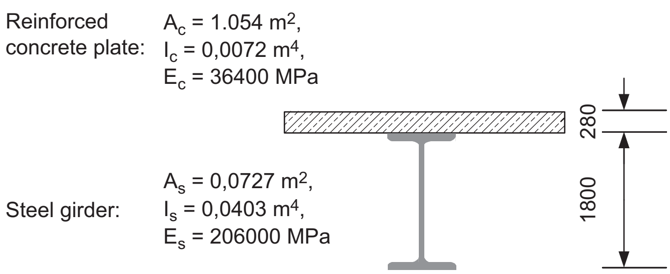

Fig. 3. Design characteristics of the girders cross-section.

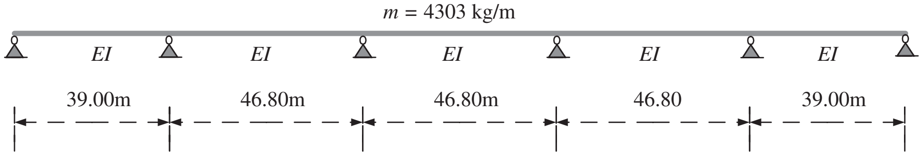

Fig. 4. Static model of the longitudinal beam of the highway bridge.

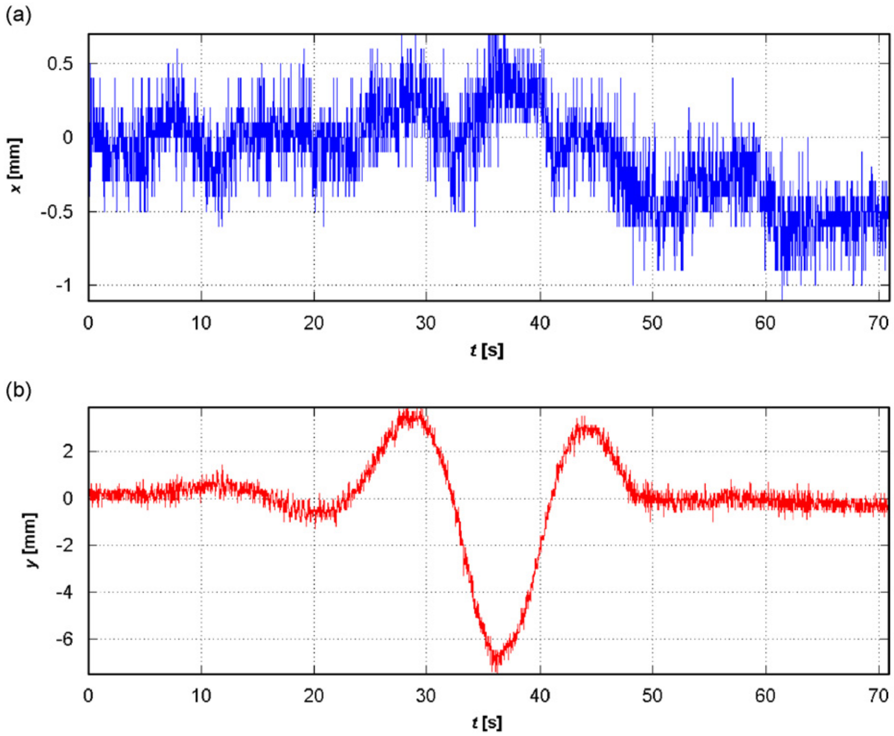

Fig. 5. The measured displacements caused by two trucks moving in parallel with $v = 20\,\text{km/h}$. x denotes the horizontal displacement (parallel to the bridge axis) and y the vertical displacements (perpendicular to the bridge axis).

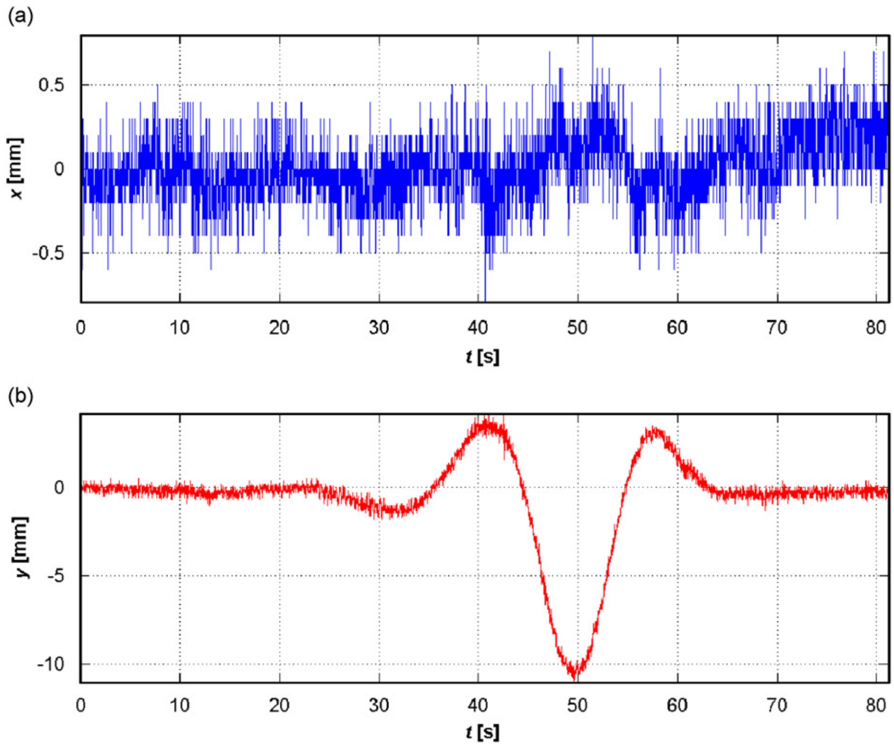

Fig. 6. The displacements caused by two trucks moving one by one with $v = 20\,\text{km/h}$. x denotes the horizontal displacement (parallel to the bridge axis) and y the vertical displacements (perpendicular to the bridge axis).

7. Summary and conclusions

An approach for the identification of a structure's parameters based on measurements of displacements caused by a short-term and moving short-term load has been presented. The proposed algorithm can be a good tool in the case when the structure, for example a highway bridge, should not be switched off from the use for a long time. The results obtained need to be verified by other identification methods and can also complete the results obtained in another way. It should be outlined that the proposed technique of stiffness identification does not take into account the interaction between the structure and the load and that the expressions presented in chapters 2–4 are given for the one-span structures.

For beam bridges, it is an important problem to determine the transverse load distribution onto particular beams. It is worth noting that the proposed method allows to identify not only the structure's parameters but also the transversal distribution of the load. The latter depends on the knowledge about the structure and the measurements we can obtain.

Such a distribution can be determined by measuring the vibrations of each beam under a load, which, on each particular beam, is proportional to the quantity (17).

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